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Topic: **Numerical Differentiation (Unit: 3)**

Numerical Differentiation:

(A) Derivatives based on Newton's forward interpolation formula:

If we want to find derivatives $y'(x)$ & $y''(x)$ *or* $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ for the values of x which lies at the top of the table, we use following formula

- For non-tabulated values of x take, $p = \frac{x-x_k}{h}$

(1) $y'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y_k + \frac{2p-1}{2!}\Delta^2 y_k + \frac{3p^2-6p+2}{3!}\Delta^3 y_k + \frac{4p^3-18p^2+22p-6}{4!}\Delta^4 y_k + \cdots\right]$

(2) $y''(x) = \frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_k + \frac{6p-6}{3!}\Delta^3 y_k + \frac{12p^2-36p+22}{4!}\Delta^4 y_k + \cdots..\right]$

- For tabulated values of x take, $p = 0$

(1) $y'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y_k - \frac{1}{2}\Delta^2 y_k + \frac{1}{3}\Delta^3 y_k - \frac{1}{4}\Delta^4 y_k + \cdots..\right]$

(2) $y''(x) = \frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_k - \Delta^3 y_k + \frac{11}{12}\Delta^4 y_k + \cdots..\right]$

(B) Derivatives based on Newton's backward interpolation formula:

We use following formula when the value of x lie at the bottom of the table.

- For non-tabulated values of x take, $q = \frac{x-x_n}{h}$

(3) $y'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2q+1}{2!}\nabla^2 y_n + \frac{3q^2+6q+2}{3!}\nabla^3 y_n + \frac{4q^3+18q^2+22q+6}{4!}\nabla^4 y_n + \cdots\right]$

(4) $y''(x) = \frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{6q+6}{3!}\nabla^3 y_n + \frac{12q^2+36q+22}{4!}\nabla^4 y_n + \cdots..\right]$

- For tabulated values of x take, $q = 0$

(1) $y'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots..\right]$

(2) $y''(x) = \frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots..\right]$